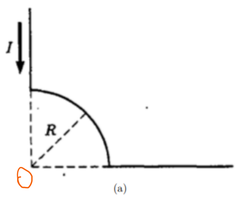
Guía #5: Campo magnético.

19 de Agosto de 2021

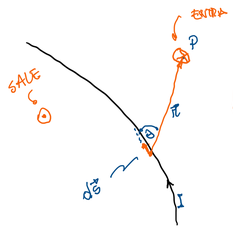
P1. Campo Magnético

Pueden encontrar videos útiles sobre este tema en este link:
`https://www.youtube.com/playlist?list=PLnA7OD6pPVGzji9lK1DtDaGiZBM_npvCT`

a) El segmento de alambre en la figura 1a conduce una corriente $I = 5A$, donde el radio del arco circular es $R = 3cm$. Determine la magnitud y dirección del campo magnético en el origen.



(a)



$$d\vec{s}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 0 \\ 5 & 1 & -4 \end{vmatrix} = \hat{i}\begin{vmatrix} -3 & 0 \\ 1 & -4 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 0 \\ 5 & -4 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = 12\hat{i} + 8\hat{j} + 17\hat{k}$$

$d\vec{s} = 2\hat{i} - 3\hat{j}$

$\vec{r} = 5\hat{i} + \hat{j} - 4\hat{k}$

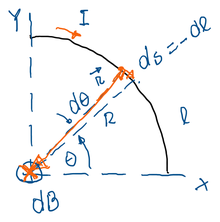
$\hat{i}\times\hat{j} = \hat{k}$ $\quad \hat{j}\times\hat{i} = -\hat{k}$

$\hat{j}\times\hat{k} = \hat{i}$

$\|d\vec{s}\times\vec{r}\| = ds\, r \operatorname{sen}\theta$

Solución:

El campo magnético en el origen, de los alambres rectos, es cero.



$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\, d\vec{s}\times\hat{r}}{r^2}$$

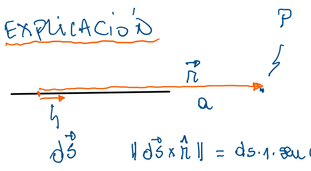
La magnitud de dB es

$$dB = \frac{\mu_0 I}{4\pi}\cdot\frac{ds \operatorname{sen} 90^\circ}{r^2} = \frac{\mu_0 I}{4\pi}\cdot\frac{dl}{R^2} = \frac{\mu_0 I}{4\pi}\cdot\frac{R\,d\theta}{R^2}$$

Integramos:

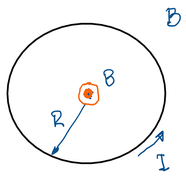
$$B = \frac{\mu_0 I}{4\pi R^2}\cdot\int_0^{\pi/2} R\, d\theta = \frac{\mu_0 I}{4\pi R^2} R\cdot\frac{\pi}{2} = \frac{\mu_0 I}{4\pi R}\cdot\frac{\pi}{2} = \frac{\mu_0 I}{8R}$$

EXPLICACIÓN

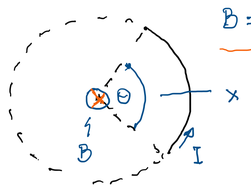


$\|d\vec{s}\times\hat{r}\| = ds\cdot 1\cdot \operatorname{sen} 0^\circ = 0$ $\quad \therefore B = 0$

OBS:



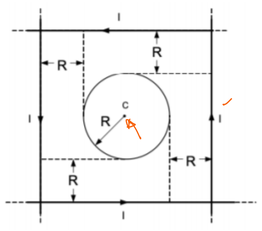
$$B = 4\left(\frac{\mu_0 I}{8R}\right) = \frac{\mu_0 I}{2R}$$



$$B = \frac{\mu_0 I}{4\pi R}\theta$$

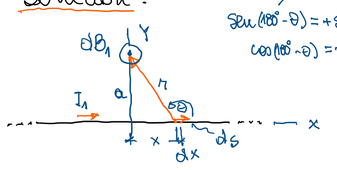
b) En la figura se aprecian líneas infinitas a través de las cuales fluyen corrientes de igual magnitud I y sentidos indicados en la figura 1b. En medio de las líneas se encuentra una espira circular de radio R, a través de la cual fluye una corriente I'. Determina el sentido y la magnitud de I' para que el campo magnético en el centro de la espira (C) sea igual a cero.

Resp: $I' = I\frac{2}{\pi}$.



(b)

Solución:



$\operatorname{sen}(180^\circ - \theta) = +\operatorname{sen}\theta$

$\cos(180^\circ - \theta) = -\cos\theta$

$$dB_1 = \frac{\mu_0}{4\pi}\cdot\frac{I_1\, ds \operatorname{sen}\theta}{r^2} = \frac{\mu_0 I_1}{4\pi}\cdot\frac{dx\, a}{r^3} = \frac{\mu_0 I_1}{4\pi}\cdot\frac{a\, dx}{(x^2+a^2)^{3/2}}$$

Con: $\operatorname{sen}\theta = \frac{a}{r}$

$ds = dx$

$r = \sqrt{x^2+a^2}$

$$\therefore B_1 = \frac{\mu_0 I_1 a}{4\pi}\int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)^{3/2}} =$$

$$B_1 = \frac{\mu_0 I_1 a}{4\pi}\cdot 2\int_0^{\infty}\frac{dx}{(x^2+a^2)^{3/2}}$$

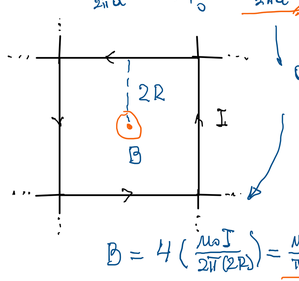
Sea $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$

Si $x = 0 \Rightarrow \theta = 0$

Si $x \to \infty \Rightarrow \theta \to \frac{\pi}{2}$

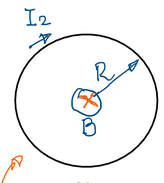
$$\therefore B_1 = \frac{\mu_0 I_1 a}{2\pi}\cdot\int_0^{\pi/2}\frac{a\sec^2\theta\, d\theta}{(a^2\tan^2\theta + a^2)^{3/2}} = \frac{\mu_0 I_1}{2\pi a}\int_0^{\pi/2}\frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^{3/2}} = \frac{\mu_0 I_1}{2\pi a}\int_0^{\pi/2}\cos\theta\, d\theta =$$

$$= \frac{\mu_0 I_1}{2\pi a}(\operatorname{sen}\theta)\Big|_0^{\pi/2} = \frac{\mu_0 I_1}{2\pi a}$$



$a = 2R$

$$B = 4\left(\frac{\mu_0 I}{2\pi(2R)}\right) = \frac{\mu_0 I}{\pi R}$$



Del problema a) con $\theta = 2\pi$ tenemos:

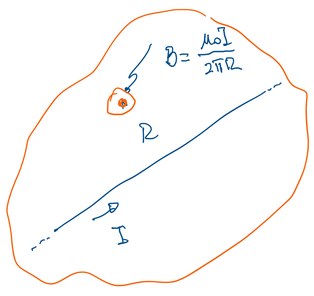
$$B_2 = \frac{\mu_0 I_2}{4\pi R}(2\pi) = \frac{\mu_0 I_2}{2R}$$

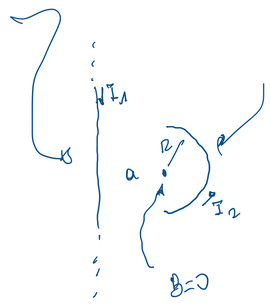
El campo magnético en el origen es cero si se cumple:

$B = B_2$

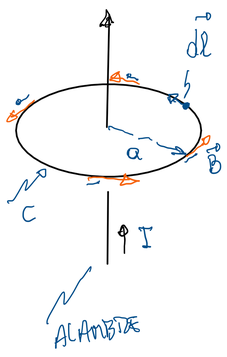
$$\frac{\mu_0 I}{\pi R} = \frac{\mu_0 I_2}{2R}$$

$$I_2 = I' = \frac{2I}{\pi}$$





LEY DE AMPÈRE



$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$$

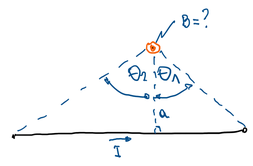
$$\oint_C B\, dl = \mu_0 I$$

$$B\oint_C dl = \mu_0 I$$

$$B(2\pi a) = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

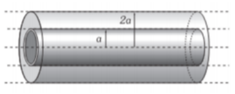
Ejemplo:



c) La figura nos representa dos hilos rectos, largos, coaxiales, aislados y metálicos. Por el hilo interior de radio a, circula una corriente I distribuida uniformemente, mientras que por el que lo rodea, de radio interno a y externo 2a, circula una corriente igual y opuesta, distribuida uniformemente por toda su sección recta, como se ilustra en la figura 1c. Calcula la magnitud del campo magnético en un punto que dista del eje en r, cuando.

1) $0 < r < a$.
2) $a < r < 2a$.
3) $r > 2a$.

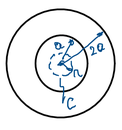
Resp 1: $B = \frac{\mu_0 I r}{2\pi a^2}$
Resp 2: $B = \frac{\mu_0 I}{2\pi r}\frac{4a^2 - r^2}{3a^2}$
Resp 3: $B = 0$



(c)

Solución:

1) $0 < r < a$:



De $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_0$ con

$$\frac{I_0}{\pi r^2} = \frac{I}{\pi a^2} \Rightarrow I_0 = \frac{r^2}{a^2} I$$

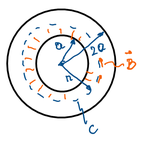
Tenemos:

$$\oint_C B\, dl = \mu_0 \frac{r^2}{a^2} I$$

$$B\oint_C dl = \mu_0 \frac{r^2}{a^2} I \Rightarrow B(2\pi r) = \mu_0 \frac{r^2}{a^2} I$$

$$B = \frac{\mu_0 r I}{2\pi a^2}$$

2) $a < r < 2a$:



La corriente por el alambre exterior, al interior de la curva C.

$$\frac{I_1}{\pi(r^2 - a^2)} = \frac{I}{\pi[(2a)^2 - a^2]} \Rightarrow I_1 = \frac{(r^2 - a^2)I}{3a^2}$$

La corriente total por el alambre compuesto es:

$$I_0 = I - I_1 = I\left(1 - \frac{r^2 - a^2}{3a^2}\right) = I\left(\frac{3a^2 - r^2 + a^2}{3a^2}\right) = \left(\frac{4a^2 - r^2}{3a^2}\right)I$$

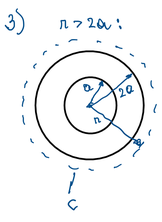
Aplicamos la ley de Ampère:

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_0 \Rightarrow \oint_C B\, dl = \mu_0\left(\frac{4a^2 - r^2}{3a^2}\right)I.$$

$$B(2\pi r) = \mu_0\left(\frac{4a^2 - r^2}{3a^2}\right)I$$

$$\therefore B = \frac{\mu_0}{2\pi r}\left(\frac{4a^2 - r^2}{3a^2}\right)I$$

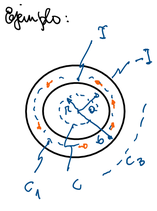
3) $r > 2a$:



En este caso $B = 0$ pues:

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_0 = \mu_0(I - I) = 0$$

Ejemplo:



1) $r < a$:

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0(0) \Rightarrow B = 0$$

2) $a < r < b$:

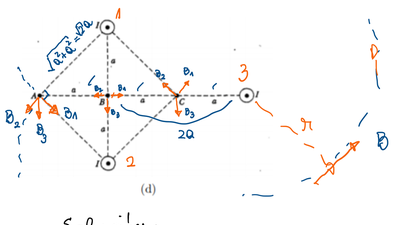
$$\oint_{C_1} \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi r) = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

3) $r > b$:

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0(I - I) = 0 \Rightarrow B = 0$$

d) Tres largos conductores paralelos llevan corrientes de $I = 2A$. La figura 1d es una vista de los extremos de los conductores, con cada una de las corrientes saliendo de la página. Si $a = 1cm$, determine la magnitud y dirección del campo magnético en los puntos A, B y C.

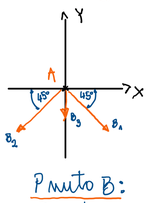


(d)

$$B = \frac{\mu_0 I}{2\pi r}$$

Solución:

Punto A



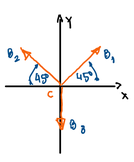
$$\vec{B}_A = 0\hat{i} - (2B_1\operatorname{sen}45^\circ + B_3)\hat{j} = 0\hat{i} - \left(\frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{6\pi a}\right)\hat{j} = 0\hat{i} - \frac{\mu_0 I}{2\pi a}\left(1 + \frac{1}{3}\right)\hat{j} = -\frac{2\mu_0 I}{3\pi a}\hat{j}$$

Donde $B_1 = \frac{\mu_0 I}{2\pi\sqrt{2}a}$, $B_3 = \frac{\mu_0 I}{2\pi(3a)} = \frac{\mu_0 I}{6\pi a}$

Punto B:

En este caso: $\vec{B}_B = \vec{B}_3 = -\frac{\mu_0 I}{2\pi(2a)}\hat{j} = -\frac{\mu_0 I}{4\pi a}\hat{j}$

Punto C:



$$\vec{B}_C = 0\hat{i} + (2B_1\operatorname{sen}45^\circ - B_3)\hat{j} = \left(2\frac{\mu_0 I}{2\pi\sqrt{2}a}\cdot\frac{\sqrt{2}}{2} - \frac{\mu_0 I}{6\pi a}\right)\hat{j} = \frac{\mu_0 I}{2\pi a}(1 - 1)\hat{j} = \vec{0}$$

Donde: $B_1 = \frac{\mu_0 I}{2\pi(\sqrt{2}a)} = \frac{\mu_0 I}{2\pi\sqrt{2}a}$; $B_3 = \frac{\mu_0 I}{2\pi a}$

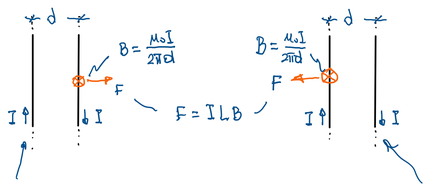
P2. Preguntas Conceptuales

a) Explique por qué dos alambres paralelos que conducen corrientes en direcciones opuestas se repelen entre sí.

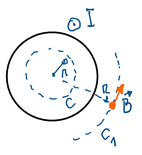
b) Un tubo de cobre hueco conduce una corriente a través de su longitud. ¿Por qué $B = 0$ dentro del tubo?¿B es diferente de cero afuera del tubo?

c) ¿Es uniforme el campo magnético creado por una espira de corriente? Explique.

(a)



(b)

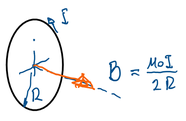


$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0(0) = 0 \Rightarrow B = 0$$

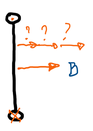
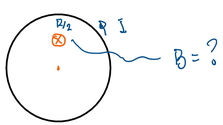
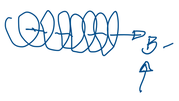
$$\oint_{C_1} \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B(2\pi R) = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi R}$$

(c)



$$B = \frac{\mu_0 I}{2R}$$



no es uniforme.